Finanziato dall'Unione Europea - NextGenerationEU a valere sul Piano Nazionale di Ripresa e Resilienza (PNRR) – Missione 4 Istruzione e ricerca – Componente 2 Dalla ricerca all'impresa - Investimento 1.1, Avviso Prin 2022 indetto con DD N. 104 del 2/2/2022, dal titolo PRIN 2022_TOZZA "Greedy Optimal Sampling for Solar Inverse Problems (GOSSIP)" codice proposta 2022HFB32T, CUP. J53C24002570001

<u>Progetto di ricerca</u> correlato all'assegno

The research activity is related to the recently funded PRIN 2022 project entitled "Greedy Optimal Sampling for Solar Inverse Problems (GOSSIP)", prot. 2022HFB32T, and will cover topics of interest to the Numerical Analysis research area, such as numerical methods for partial differential equations, inverse problems, and image processing. More precisely, the project will focus, on the one hand, on the study of high-order numerical schemes for nonlinear partial differential equations (e.g., high-order "filtered" schemes for equations of Hamilton-Jacobi type), and, on the other hand, on numerical solution of problems in the area of image processing, with more emphasis on astronomical imaging.

Below more details on the main research topics and expected goals.

In general, the solution of nonlinear partial differential equations (PDEs) of Hamilton-Jacobi type is not classical (belonging to C^1 class), independently on the regularity of the initial condition, since its solutions may develop singularities in finite time. Consequently, it was necessary to introduce some appropriate definition of solution in weak sense. This was done by Crandall and Lions in 80s, introducing the definition of viscosity solution. The theory of viscosity solutions allows to obtain existence and uniqueness in a wide range of situations, and that kind of solution is adopted in different and interesting application fields, like e.g. image processing, optimal control, differential games, front propagation, etc. The lack of smoothness of viscosity solutions makes it difficult to develop efficient approximations, that is, accurate results at a reasonable computing cost.
We look for non-monotone schemes in order to get high-order accurate schemes, which also allow us to adopt coarser grids. A goal in that part of the project is to develop strategies for detecting the singularities that may occur in finite time in evolutive problems. Moreover, a theoretical analysis of those schemes in terms of stability and convergence is also expected.

Such a theoretical study finds application in the field of astronomical imaging.
Astronomical images are of crucial importance for astronomers since they contain a lot of information about celestial bodies that cannot be directly accessible. Most of the information available for the analysis of these objects starts with sky explorations via telescopes and satellites (and there are growing interests in the construction of more compact and possibly cheaper satellites, see e.g. the European Space Agency - ESA website for a general overview). Unfortunately, the quality of astronomical images is usually very low with respect to other real images and this is due to technical and physical features related to their acquisition process. For those particular features, the astronomical images require a careful adaptation of the techniques used in image processing, e.g. for image segmentation, source deblending, denoising, 3D shape recover, deblurring and super-resolution. A goal in that part of the project is to analyze and compare different numerical techniques in order to select which one can give better results, and the adaptations needed to obtain accurate results at a reasonable computing cost for problems related to astronomical imaging.

One of the final goal is to use the adaptivity strategy of high-order "filtered" schemes for PDEs, developed for detecting the singularities, in the context of astronomical images, in particular images related to solar flares recorded by STIX and, since STIX has been only recently launched, solar flares recorded by the NASA satellite RHESSI, active from February 2002. In addition, robust level-set methods that provide useful information on the image gradient (edge detection) could be used. The strategies developed in the context of nonlinear PDEs will be in this way combined with the different skills that involve the other two research units belonging to the PRIN project above mentioned, in a heterogeneous and complementary work environment.

The new Fellow will work under the supervision of Prof. Silvia Tozza, PI of the UNIBO research unit involved in the PRIN project above mentioned.